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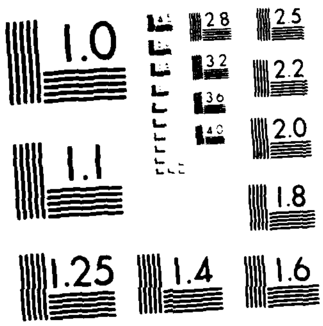

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Chapel Hill, North Carolina

DECOMPOSABILITY OF p-CYLINDRICAL MARTINGALES

by

Z. Suchanecki
and
A. Weron

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A class of p-cylindrical martingales in locally convex spaces is studied. We obtain a general form of convergent p-cylindrical martingales in barrelled spaces. Using the locally convex space technique, new results even in Banach spaces are deduced. It is proved that for $p \geq 1$ the adjoint to p-absolutely summing operator is p-decomposing for any p-cylindrical martingale.

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DECOMPOSABILITY OF p-CYLINDRICAL MARTINGALES

by

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Summary: A class of p-cylindrical martingales in locally convex spaces is studied. We obtain a general form of convergent p-cylindrical martingales in barrelled spaces. Using the locally convex space technique, new results even in Banach spaces are deduced. It is proved that for $p \geq 1$ the adjoint to p-absolutely summing operator is p-decomposing for any p-cylindrical martingale.

Key words: cylindrical martingale, cylindrical random element, p-absolutely summing operator.

1980 AMS Subject Classification: Primary 60G48; Secondary 60B11, 47B10.

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0. Introduction.

The aim of this paper is to examine the class of p-cylindrical martingales in locally convex spaces. We study the relation between integrability and τ-continuity. Our general approach leads us to new results even in Banach spaces. One of the main results concerns the decomposability of cylindrical martingales in Banach spaces - Th. 2.2. The second part of this theorem is a cylindrical version of the recent result of Chobanjan, Linde and Tarieladze [3] and may be considered as a new property of p-absolutely summing operators.

Metivier and Pellaumail ([4] and [5], Chapter 6) pointed out that it is possible to develop the theory of stochastic integration with respect to 2-cylindrical martingales in Banach spaces, cf. also [8]. The important examples are cylindrical Brownian motion and white noise in time and in space. Such processes have been discussed in connection with quantum field theory, theory of *partial differential* equations involving random terms and filtering theory in electrical engineering, cf. for example [7] and references therein. This motivated our study.

The plan of the paper is as follows. In the rest of this section we set up the basic notation and conventions. Section 1 is devoted to cylindrical random elements in locally convex spaces. Sometimes such elements are called generalized processes. The relation between integrability and τ-continuity of cylindrical random elements is established. It is an extension of Chatterji's results obtained in [1] for random variables. Also the important problem of decomposability of cylindrical random elements is studied. Th. 1.1 is an addendum to Kwapien's theorem ([2], th. 2). Namely, using the locally convex space technique we obtain the decomposition of XS^* by a G-valued random variable instead of G''-valued random variable, where F,G are Banach spaces, $X:F'_\tau \to L^p$ is a cylindrical random element and $S:F\to G$ is a p-absolutely summing operator.

The point is that, in contrast with other papers, we consider here τ-continuous cylindrical random elements.

In section 2 we consider the class of p-cylindrical martingales, defined as a family of cylindrical random elements. Th. 2.1 gives the general form of convergent p-cylindrical martingales, if F'_τ is a barrelled space and $p > 1$. Th. 2.2 gives a condition under which some p-cylindrical martingales are decomposable by convergent vector valued martingales in Banach spaces.

In the paper we will use the following notation. The capital letters F,G will denote vector spaces and f,g their elements, respectively. For the detailed definitions and preliminary facts on vector spaces we refer the readers to [11]. The symbol $\langle F,G\rangle$ will denote the dual pair and $\langle\ ,\ \rangle$ its bilinear form on $F \times G$. For $A \subset F$ by A° denote the polar of A. If F is a locally convex space /l.c.s./ then F; denotes its topological dual and $\langle f,f'\rangle$ or $\langle f',f\rangle$ stands for the value of a functional f' at a point $f \in F$. By F_σ and F_τ we will denote the space F with the weak topology $\sigma(F,G)$ and with the Mackey topology $\tau(F,G)$, respectively. If F is a Banach space then by F without any additional subscript we mean the space F with its norm topology. If S is a linear operator S* will stand for its adjoint.

1. Cylindrical random elements. Let $(\Omega,\mathcal{B},P)$ be a probability space and $L^p = L^p(\Omega,\mathcal{B},P)$, $p \geq 0$. Let $\langle F,G\rangle$ be a dual pair.

DEFINITION. Any linear operator $X\colon\ G \to L^p$ is called a p-*cylindrical random element* (p-c.r.e.), $p \geq 0$.

If F is a locally convex space and $G = F'$, then any map $x\colon\ \Omega \to F$, which is weakly measurable, F-valued random variable, defines a 0-c.r.e. X by the following formula

$$X f' = \langle x,f'\rangle\ ,\quad f' \in F'\ .$$

If additionally, x has a p-weak order, $p>0$, (i.e., $|\langle x,f'\rangle|^p$ is integrable), then X is a p-g.r.e. The converse implication is not true in general; take $\;= [0,1]$, $F = L^2[0,1]$ and $X = Id\colon\ L^2[0,1] \to L^2[0,1]$.

DEFINITION. Let $p \geq 1$. We say that a p-c.r.e. X is *integrable* if for each $B \in \mathcal{B}$ there exist $f_B \in F$ such that

$$\langle f_B, g\rangle = \int_B Xg\, dP \quad \text{for each } g \in G\ .$$

The most interesting case for our study is when F is locally convex and $G = F'$.

PROPOSITION 1.1 ([12], Th. 1)

Let X be a 1-c.r.e. on a l.c.s. F. Then

(a) If $X\colon\ F'_\tau \to L^1$ is continuous, then X is integrable.

(b) If F is sequentially complete and X is integrable, then $X\colon\ F'_\tau \to L^1$ is continuous.

COROLLARY 1.1

Let F be a complete l.c.s. and X be a p-c.r.e., $p > 1$, then the following conditions are equivalent:

(i) $X\colon\ F'_\tau \to L^p$ is continuous.

(ii) X is integrable.

Proof. (i) => (ii). It follows from Prop. 1.1(a). (ii) => (i). By Prop. 1.1 (b) X is continuous as a mapping $X\colon\ F'_\tau \to L^1$; consequently $X\colon\ F'_\tau \to L^o$ is also continuous. Since $X(F') \subset L^p$ then by ([15], Th. 1) we have that $X\colon\ F'_\tau \to L^p$ is continuous.

REMARK. Chatterji in [1] studied first the relation between Pettis integrability of vector valued functions (random elements) and continuity of the corresponding c.r.e.. In particular, he has shown that if F is a complete l.c.s. then $x\colon\ \Omega \to F$ is Pettis integrable iff the corresponding c.r.e. $X\colon\ F'_\tau \to L^1$ is continuous and $X(U^o)$ is relatively weak compact in L^1 for each neighborhood U of zero in F. This result gives the following, well known in Banach spaces, fact: each weakly measurable F-valued function with finite p-order, $p > 1$, is Pettis integrable.

We are now going to consider the situation for c.r.e.'s which are not necessarily defined by random elements.

PROPOSITION. 1.2

Let F be a sequentially complete l.c.s. and let X be a 1-c.r.e. Then

(a) *If X is integrable, then* $X(U^o)$ *is relatively weak compact in* L^1 *for each neighborhood U of zero in F.*

(b) If additionally F is a reflexive Banach space, then the converse implication holds.

Proof. If U is an absolutely convex neighborhood of zero in F, then U^o is $\sigma(F',F)$ compact ([11], III, 4.3 and IV, 1.4). Thus we have part (a).

(b) If $X(B^o)$ is weakly relatively compact in L^1, where B is the unit ball in a reflexive Banach space F, then X is bounded. Since F is reflexive $F'_T = F'$ (cf. [11], IV, 5.5). Thus $X: F' \to L^1$ is continuous and consequently it is integrable.

COROLLARY. 1.2

Each p-c.r.e. X on a reflexive Banach space is integrable for $p > 1$.

Proof. By Proposition 1.2 it is enough to check that $X(B^o)$ is relatively weak compact in L^1. Note that by the closed graph theorem

$$\sup_{f' \in B^o} \int_\Omega |X f'|^p \, dP < \infty .$$

Consequently by the Vallèe-Poussin theorem (see [6], II, T22) the family $\{Xf', f' \in B^o\}$ is relatively weak compact in L^1.

DEFINITION. (cf. [2]). Let F be a Banach space. We say that a c.r.e. $X: F'_T \to L^p$ is *p-decomposable*, $p > 0$, if there exists a random element $x: \Omega \to F$ such that

$$1^o \quad X f' = \langle x, f' \rangle \quad \text{for each } f' \in F'$$

$$2^o \quad \int_\Omega \|x\|^p \, dP < \infty .$$

As we have pointed out before even in the Hilbert space $L^2[0,1]$ there are c.r.e.'s which are not 2-decomposable. But in the case of Hilbert spaces it is easy to find a full characterization. Namely, if F is a separable Hilbert space, then a 2-c.r.e. $X: F' \to L^2$ is 2-decomposable iff X is a Hilbert-Schmidt operator (see [5], p. 177). In the case of Banach spaces the situation is much more difficult.

The next result gives a condition for p-decomposability of c.r.e. It seems to be an interesting addendum to the Kwapien's theorem ([2], Th. 2) and will be crucial for the proof of Th. 3.2. Let us recall that an operator $S : F \to G$ is p-*absolutely summing* iff

$$/x/ \quad \|Sf\|^p \le \pi_p^p(S)\int_{B^o} |\langle f,f'\rangle|^p \, d\mu(f') \quad \text{for all } f \in F ,$$

where μ is a Radon measure on the unit ball B^o of F' equipped with the $\sigma(F',F)$ topology, (cf. [9], 17.3.2).

THEOREM. 1.1

Let F,G be Banach spaces and $S: F \to G$ be a p-absolutely summing operator. If $X: F'_\tau \to L^p$ is a continuous c.r.e., then

(i) $Y = X S^*$ is p-decomposable by a G-valued function y.

(ii) Let L^p be a separable space, then we have the following inequality

$$\int_\Omega \|y(\cdot)\|^p \, dP \le \pi_p^p(S)\int_{B^o}\|Xf'\|^p_{L^p} \, d\mu(f') ,$$

where B^0 and μ are as in the Pietsch inequality /x/.

Proof. From the definition of τ-continuity of X there exists an absolutely convex and weakly compact subset $A \subset F$ such that $X(A^o)$ is bounded in L^p. The continuity of S implies (cf. [11], p. 158) that $S^*: G'_\tau \to F'_\tau$ and thus there exists an absolutely convex and weakly compact subset $D \subset G$ such that $S^*(D^o) \subset A^o$. Using the Schaefer's convention ([11], III §7) let us introduce the auxiliary Banach spaces

$$F'_{A^o} = \hat{F}'_\tau / p^{-1}_{A^o}(0) \quad , \quad G'_{D^o} = \hat{G}'_\tau / p^{-1}_{D^o}(0) \quad ,$$

$$F_A = \bigcup_{n=1}^{\infty} nA \quad , \quad G_D = \bigcup_{n=1}^{\infty} nD \quad ,$$

with norms defined by the corresponding Minkowski's functionals. Let

$$\phi_{A^o}: \ F'_\tau \to F'_{A^o} \quad \text{and} \quad \phi_{D^o}: \ G'_\tau \to G'_{D^o}$$

denote the cannonical quotient mappings and let

$$\psi_A: \ F_A \to F \quad \text{and} \quad \psi_D: \ G_D \to G$$

denote the cannonical injections. We have the following diagrams.

$$\begin{tikzcd} G'_\tau \arrow[rr, "S^*"] \arrow[dr, "\phi_{D^o}"'] & & F'_\tau \arrow[rr, "X"] \arrow[dr, "\phi_{A^o}"] & & L^p \\ & G'_{D^o} \arrow[ur, "S^*_D"] \arrow[rr, "T"] & & F'_{A^o} \arrow[ur, "X_D"'] & \end{tikzcd}$$

where S^*_D and X_D denote the induced mappings and $T = \phi_{A^o} S^*_D$. From the diagram we see that X_D and T are continuous. Let us note that T* is p-absolutely summing. Indeed, it follows from ([11], III Ex. 3c and IV §2) that $(F'_{A^o})' = F_A$, $(G'_{D^o})' = G_D$ and $\phi^*_{A^o} = \psi_A$. Thus $T^*: \ F_A \to G_D$. Moreover the inclusion $S^*(D^o) \subset A^o$ implies $S(A) \subset D$, (cf. [11], IV, 2.3). Consequently $T^* = S\psi_A$ is p-absolutely summing. By the Kwapien's theorem ([2], th. 2) we have that $X_D T$ is p-decomposable by $y_D(\cdot): \ \Omega \to (G'_{D^o})' = G_D$. Using the continuity of the cannonical injection ψ_D we claim that the following function

$$y(\cdot) = \psi_D y_D(\cdot): \ \Omega \to G$$

decompose X S*.

The proof of part (ii) will be given in two steps. Consider first the case X: $F'_\tau \to L^p$ is finite dimensional. Since X is continuous it is p-decomposable by a function x: $\Omega \to F$. Consequently XS* is p-decomposable by $y = Sx: \Omega \to G$.

According to the assumption S is p-absolutely summing. Hence

$$\int_\Omega \|y(\omega)\|^p \, dP(\omega) = \int_\Omega \|Sx(\omega)\|^p \, dP(\omega)$$

$$\le \pi_p^p(S) \int_\Omega \int_{U^o} |\langle x(\omega), f'\rangle|^p \, d\mu(f') \, dP(\omega)$$

$$= \pi_p^p(S) \int_{U^o} \|Xf'\|^p_{L^p} \, d\mu(f') \ .$$

Now let X be any c.r.e. and assume that the space L^p is separable. Thus L^p has a Schauder basis and there exists a sequence of finite-dimensional operators X_n such that X is point approximated by X_n. Let $X_n S^*$ and XS^* be p-decomposable by $y_n(\cdot)$ and $y(\cdot)$, respectively. We have

$$\int_\Omega \|y_n(\omega) - y_m(\omega)\|^p \, P(d\omega)$$

$$\le \pi_p^p(S) \int_{U^o} \|(X_n - X_m) f'\|^p \, d\mu(f') \ .$$

By using the Banach-Steinhaus theorem we observe that the right side of the last inequality converges to zero as $n,m \to \infty$. Hence $y_n(\cdot)$ is convergent in $L^p(G)$. On the other hand for each $g' \in G'$ we have

$$\langle y_n(\cdot), g'\rangle = X_n S^* g' \xrightarrow{L^p} XS^* g' = \langle y(\cdot), g'\rangle$$

and consequently $y_n(\cdot) \to y(\cdot)$ in $L^p(G)$. Thus the inequality in part (ii) holds for any c.r.e. and the proof of th. 1.1 has been completed.

REMARK. The assumption L^p is separable, used in part (ii) of the above theorem, may be omitted in the following two cases:

1° X is compact

2° X is generated by a weakly measurable p-integrable function.

Indeed, in the first case we may use the metric approximation property of L^p and in the second case one may construct a needed approximation of X by using operators X_n: $F'_\tau \to L^p$ generated by simple functions.

As an illustration to the last theorem consider the following:

EXAMPLE. As we know for a Hilbert space H a 2-c.r.e. X: $H' \to L^2$ is 2-decomposable iff X is a Hilbert-Schmidt operator. Consider a linear continuous operator S: $H \to G$, where G is a reflexive Banach space. Using the fact $G' = G'_\tau$ define a new 2-c.r.e. Y in the Banach space G by the following formula:

$$Y = XS^*: \quad G'_\tau \to L^2 .$$

Then it is easy to observe that Y is 2-decomposable too. Namely, if $Xh = \langle x(\cdot),h\rangle$ for $\forall h \in H'$ then $y(\cdot) = Sx(\cdot)$ decomposes Y since for $\forall g' \in G'$ we have

$$\langle y(\cdot),g'\rangle = \langle Sx(\cdot),g'\rangle = \langle x(\cdot),S^*g'\rangle$$

$$= XS^*g' = Yg' .$$

Thus we obtain that any 2-c.r.e. Y: $G'_\tau \to L^2$ which admits a factorization through a Hilbert-Schmidt operator $Y = XS^*$, where S: $H \to G$ is linear continuous and X: $H' \to L^2$ is Hilbert-Schmidt, is 2-decomposable. But this factorization condition is equivalent to the statement that S is 2-absolutely summing (see [2], Cor. 2). Th. 1.1 shows that, in fact, the assumption "G is reflexive" is not needed!

2. <u>Cylindrical martingales</u>. In this Section we will first introduce the conditional expectation for c.r.e.'s with respect to some sub σ-field $\mathcal{A} \subset \mathcal{B}$. For this we will use the Pettis type integral which was discussed in the previous section.

Let $E^{\mathcal{A}}$: $L^1(\mathcal{B}) \to L^1(\mathcal{A})$ denote the usual operator of the conditional expectation, (see [6], §4), and let X: $F'_\tau \to L^p$, $p \geq 1$, be an integrable p-c.r.e. obtained as a composition of operators $E^{\mathcal{A}}$ and X. Note that if X: $F'_\tau \to L^p$ is continuous, then $Y = E^{\mathcal{A}}X$ is also continuous. Thus if X: $F'_\tau \to L^p$ is continuous, $1 < p < \infty$, then for each sub σ-field $\mathcal{A} \subset \mathcal{B}$ there exists a 1-c.r.e. $Y = E^{\mathcal{A}}X$: $F'_\tau \to L^1$ such that for each $A \in \mathcal{A}$ we have

$$\int_A Y\, dP = \int_A E^A X\, dP = \int_A X\, dP .$$

REMARKS. 1. Let us note that presented above extension of the notion of the conditional expectation is also useful for random elements. As we know, any random element with values in a Banach space F, $x: \Omega \to F$, which has a weak p-order defines a c.r.e. $X: F'_\tau \to L^p$ and we may put

$$E^A x = E^A X \quad \text{i.e.,} \; E^A\langle x,f'\rangle = E^A Xf' \text{ for any } f' \in F' .$$

Thus always there exists the conditional expectation in the above generalized sense. It is important since V.I. Rybakov ([10]) has given an example of Pettis integrable random element x with values in a reflexive Banach space F such that for some sub σ-field A does not exist any A-measurable F-valued function h such that

$$\int_A \langle h,f'\rangle\, dP = \int_A \langle x,f'\rangle\, dP \quad \text{for } A \in A,\; f' \in F',$$

i.e., the ordinary conditional expectation for x does not exist. Moreover it is known (see [14]) that one may construct such examples in any infinite dimensional Banach spaces. Thus even for random elements the conditional expectation exists only in the above sense.

2. Note that the space $L^1(F)$- of all Pettis integrable functions is not complete but if F is a separable Fréchet space then its completion $\hat{L}^1(F) = F \otimes_\varepsilon L^1$, (see [13], Th. 3.2).

Finally, let us present some elementary properties of the conditional expectation.

(a) $< \int_A E^A X\, dP, f'> = \int_A E^A Xf'\, dP = \int_A Xf'\, dP, \; A \in A, \; f' \in F'$.

(b) If $A \subset A'$, then $E^A(E^{A'}X) = E^A X$.

(c) If F is a Banach space, then E^A is contraction.

(d) If F is a separable Fréchet space and $X \in F\otimes_\varepsilon L^1$, then $E^A: F\otimes_\varepsilon L^1(B) \to F\otimes_\varepsilon L^1(A)$ is linear and continuous.

Now we will proceed to investigate cylindric martingales. Let us assume that F is a complete l.c.s. and T is a linear ordered set. The main results of the paper are collected in Th. 2.1 and Th. 2.2.

DEFINITION. By a p-*cylindrical process* $(X_t)_{t\in T}$ we mean a family of p-c.r.e. indexed by the parameter set T. It is integrable if for each $t \in T$, X_t is an integrable c.r.e. (cf. Section 1).

As an immediate consequence of Corollary 1.1 we have

PROPOSITION 2.1

Let $(X_t)_{t\in T}$ be a p-cylindrical process, $p > 1$, then the following conditions are equivalent:

(i) $(X_t)_{t\in T}$ is integrable.
(ii) $(X_t)_{t\in T}$ is τ-*continuous*, i.e., for each $t\in T$ X_t: $F'_\tau \to L^p$ is continuous.

DEFINITION. Let $(A_t)_{t\in T}$ be a net of sub σ-fields of $\mathcal{B}$, i.e., if $t_1 < t_2$, then $A_{t_1} \subset A_{t_2}$. A p-cylindrical process $(X_t)_{t\in T}$, $p \geq 1$, is called p-*cylindrical martingale* if

1^o $(X_t)_{t\in T}$ is adapted with respect to $(A_t)_{t\in T}$,

2^o $(X_t)_{t\in T}$ is τ-continuous,

3^o $E^{A_s}X_t = X_s$ for $s < t$.

PROPOSITION 2.2

Let $(X_t)_{t\in T}$ be a τ-continuous p-cylindrical process, then the following conditions are equivalent:

(i) $(X_t)_{t\in T}$ is a p-cylindrical martingale,

(ii) $(X_t f')_{t\in T}$ is a real martingale for each $f' \in F'$.

THEOREM 2.1

Let F'_τ be a barrelled space, $p > 1$ and let $(X_t)_{t\in T}$ be a p-cylindrical martingale with respect to $(A_t)_{t\in T}$, such that

$$\sup_{t\in T} E|X_t f'|^p < \infty \quad \text{for each } f' \in F' .$$

Then there exists a p-c.r.e. X: $F'_\tau \to L^p$ which is continuous and such that

$$X_t = E^{At} X \ , \ t \in T \ .$$

Proof. By the assumption $\sup_{t\in T} |X_t f'|^p < \infty$ for any $f' \in F'$. Therefore the set $\{X_t, t\in T\}$ forms a family of continuous linear operators from $F'_\tau \to L^p$, which is bounded in each point $f' \in F'_\tau$. Since F'_τ is barrelled space then the family $\{X_t, t\in T\}$ is equicontinuous (see [11], p. 107). On the other hand, for each $f' \in F'_\tau$ the process $(X_t f')_{t\in T}$ is a real p-integrable martingale, $p > 1$. Hence by the martingale convergence theorem ([6], VT 22) for each $f' \in F'_\tau$ there exists a random variable $\xi(\cdot) = \xi_{f'}(\cdot) \in L^p$ such that $E^{At}\xi = X_t f'$ and $E|X_t f' - \xi|^p \to 0$ as $t \to \infty$. Putting $X(f') = \xi$ we define a linear operator X: $F'_\tau \to L^p$. It is continuous as a closure point of the family $\{X_t, t\in T\}$ in the point convergent topology (cf. [11], p. 109). Finally, the equality $X_t = E^{At}X$, $t \in T$ is a consequence of the definition of the conditional expectation and the theorem is proved.

Let $p > 1$ and let $M_p(\mathbb{R})$ denote the Banach space of all p-integrable real martingales $m = (m_t)_{t\in T}$ for which the following condition holds:

$$\|m\|_X = \lim_{t\to\infty} (E|m_t|^p)^{1/p} < \infty$$

Let us observe that any p-cylindrical martingale $(X_t)_{t\in T}$ defines a linear mapping $\tilde{X}$: $F'_\tau \to M_p(\mathbb{R})$ by the formula $\tilde{X}f' = (X_t f')_{t\in T}$. If F'_τ is barrelled space and $\sup_{t\in T} E|X_t f'|^p < \infty$, then $\tilde{X}$ is continuous. Indeed, by Theorem 2.1 there exists a continuous p-c.r.e. X such that

$$\begin{aligned} \|\tilde{X}f'\|_X &= \lim_t (E|X_t f'|^p)^{1/p} \le \sup_t (E|X_t f'|^p)^{1/p} = \sup_t (E|[E^{At}X]f'|^p)^{1/p} \\ &\le \sup_t (E\, E^{At}|Xf'|^p)^{1/p} = \sup_t (E|Xf'|^p)^{1/p} \\ &= (E|Xf'|^p)^{1/p} \ . \end{aligned}$$

Thus we have proved the following.

PROPOSITION 2.3

Let F'_τ be a barrelled space and $(X_t)_{t\in T}$ be a p-cylindrical martingale for which $\sup_{t\in T} E|X_t f'|^p$ is finite for each $f' \in F'$, $p > 1$, then $\tilde{X}$: $F'_\tau \to M_p(\mathbf{R})$ is continuous.

REMARK. Metivier and Pellaumail have studied the case of 2-cylindrical martingales in Banach spaces. Note that Prop. 2.3 implies that our definition of cylindrical martingales reduces to the one used in [4], and Prop. 2.2 shows that it reduces to the second used in [5].

THEOREM 2.2

Let $1 \le p < \infty$, F, G be Banach spaces and S: $F \to G$ be a p-absolutely summing. If $(X_t)_{t\in T}$ is p-cylindrical martingale in F, then

(a) $(X_t S^*)_{t\in T}$ is p-decomposable by a G-valued martingale $(y_t)_{t\in T}$.

(b) If L^p is separable and for each $f' \in F'$ $(X_t f')_{t\in T}$ is a convergent in L^p martingale, then $(X_t S^*)_{t\in T}$ is p-decomposable by a convergent in $L^p(G)$ martingale $(y_t)_{t\in T}$.

Proof. From Theorem 1.1 (i) the proof of part (a) follows immediately.

(b) Let for each $t \in T$ the c.r.e. $X_t S^*$ be p-decomposable by y_t. Suppose "a contrario" that $(y_t)_{t\in T}$ does not converge in $L^p(G)$. Thus there exists $\varepsilon > 0$ and a sequence $t_1 < t_2 < t_3 \ldots$ of elements from T such that

$$E\|y_{t_m} - y_{t_n}\|^p > \varepsilon \quad \text{for each } m,n = 1,2,\ldots$$

On the other hand by Theorem 1.1 (ii) we have

$$E\|y_{t_m} - y_{t_n}\|^p \le \pi_p^p(S)\int_{B^o} \|X_{t_m} f' - X_{t_n} f'\|^p_{L^p}\, d\mu(f') .$$

By the assumption $(X_t f')_{t\in T}$ converges in L^p for each $f' \in F'$ and by the Banach-Steinhaus theorem the operators X_t are uniformly bounded on B^o. Thus using the dominated convergence theorem we see that the right side of the above inequality goes to zero as m and n tends to ∞. This gives the needed contradiction and thus completes the proof.

Using the last remark from Section 1, we obtain as an immediate corollary the recent result of Chobanjan, Linde and Tarieladze.

COROLLARY 2.1 ([3], p. 138).

Let $1 \le p < \infty$, F,G be Banach spaces and S: $F \to G$ be p-absolutely summing. If $(x_t)_{t\in T}$ is an F-valued process such that for each $f' \in F'$ $\langle x_t, f'\rangle_{t\in T}$ is a convergent in L^p martingale, then $(Sx_t)_{t\in T}$ is a convergent in $L^p(G)$ martingale.

We conclude with an example showing why the decomposability problem is important for cylindrical martingales.

EXAMPLE. Consider F = H is a Hilbert space. Then a 2-cylindrical martingale Y_t: $H' \to L^2$ is 2-decomposable iff the mapping $H' \ni h \to Y_t h \in L^2$ is a Hilbert-Schmidt operator for any $t \in T$, i.e., $Y_t \in HS(H', \mathcal{M}_2(\mathbf{R}))$. Thus there exists the correspondence between $Y_t \in HS(H', \mathcal{M}_2(\mathbf{R}))$ and an H-valued martingale $y_t \in \mathcal{M}_2(H)$ which is an isometry. In a similar way if G is another Hilbert space the mapping (defined in [5])

$$X \to \int X \, dY$$

is an isometry from the space of all L(H,G)-valued processes which are square integrable with relation to 2-cylintrical martingale Y into the space of square integrable G-valued martingales (cf. [5], p. 185). This means that stochastic integral $\int X \, dY$ considered as a 2-cylindrical martingale is 2-decomposable by a G-valued martingale.

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